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The Analysis of the Geomagnetic Secular Variation

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THE ANALYSIS OF THE GEOMAGNETIC SECULAR VARIATION

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After some discussion of the general properties of the secular variation field, a graphical method is described of analyzing it in terms of separate dipoles of arbitrary direction. The analysis shows that the major part of the field for epoch 1922.5 is explained by about twelve *vertical* dipoles below the surface of the core. The relation of this result with other analyses by McNish (1940) and Bullard (1948) is discussed.

The depth of the dipoles confirms that the origin of the secular variation must lie in the core of the earth; because of the high electrical conductivity of the core material it must in fact be due to a thin current sheet at the surface of the core, and this interpretation also gives an explanation for the existence of only vertical sources.

The presence of only vertical sources, and in particular the presence of several near the equator, does not support the existence of the toroidal field, which is an essential step in Bullard's (1949) dynamo theory of the main field.

1. Introduction

The geomagnetic field (**F**), which is known at many points on the earth's surface at various epochs, undergoes a secular change which may be as much as 150γ per year ($1\gamma = 10^{-5}$ gauss) in magnitude and 15 min. of arc per year in direction. The vector $\dot{\mathbf{F}}$ describing this secular change is a complicated function of latitude and longitude and changes slowly with time. A very complete survey of the secular variation between the years 1905 and 1945 is available (Vestine, Laporte, Cooper, Lange & Hendrix 1947*a*), containing world charts showing the lines of equal annual change of the various elements *X*, *Y*, *H*, *Z*, *F*, *I* and *D**, plotted for the epochs 1912.5, 1922.5, 1932.5 and 1942.5. Figures 1 and 2 give the $\dot{H}$ and $\dot{Z}$ charts for 1922.5. From these and earlier data the following properties of the secular change can be deduced:

(i) The lines of equal change in any one element, called isopors, form a series of oval-shaped closed curves surrounding points, termed isoporic foci, at which the changes are most rapid.

(ii) At any epoch such sets of isopors cover areas of continental size, which will be termed active areas, and are separated by areas over which the changes are small, which will be termed quiescent areas.

(iii) The isopors in a region may change considerably in form in the course of a few decades, and active areas may decay or establish themselves during that time.

(iv) Each isoporic focus and its associated set of isopors drifts to the west at a mean rate of about 0·3° of longitude per year.

* *X*, *Y* and *Z* are respectively the field components in the north, east and vertically downward directions, *H* is the total horizontal component, *F* the total field intensity, *I* the angle of inclination or dip and *D* the angle of declination.

The field $\dot{\mathbf{F}}$ can be separated by spherical harmonic analysis into a part of external origin $\dot{\mathbf{F}}_e$ and a part of internal origin $\dot{\mathbf{F}}_i$ (Chapman & Bartels 1940, vol. 2, chap. 17). This method of analysis of the secular variation field $\dot{\mathbf{F}}$ shows that it is wholly of internal origin (Bartels 1925; Vestine, Laporte, Lange & Scott 1947*b*). It does not, however, assist in relating the secular change to likely physical causes within the earth; for it expresses the scalar potential of the field as a series of solid harmonics, each being the potential of a dipole or multipole

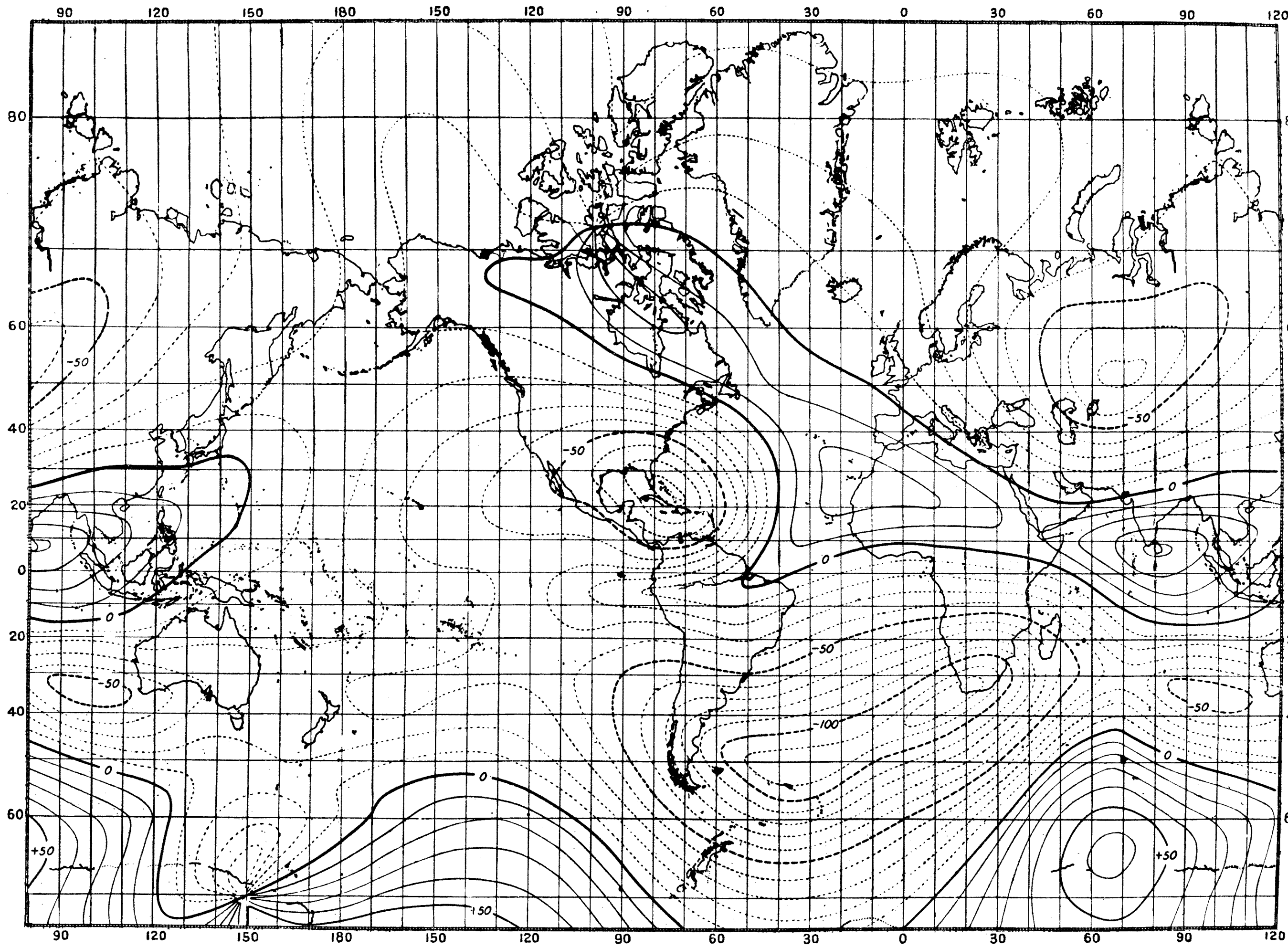

FIGURE 1. Geomagnetic secular change in gammas per year, horizontal intensity, epoch 1922.5.

at the geocentre. Because the secular variation has such a short time scale compared to geological processes, Elsasser (1939, 1946) and Bullard (1948) suggested that its origin lay in motions in the fluid core of the earth which is likely to be an electrical conductor. These motions in the core are likely to be eddies whose axis may be remote from the geocentre. Whatever are the physical processes associated with these eddies which cause electric currents to flow, it is to be expected that the external magnetic fields due to these circuits would be most simply represented by dipoles, or simple multipoles, near the centre of the eddy. The simplest source* of an active area of the secular variation would be such a dipole, whose

* Though the strength of such a source is expressed in units of gauss cm.^{3}/year, it will be convenient to speak of the dipole responsible for the secular variation, the time variation being understood from the context.

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magnitude or direction was changing with time. Thus it could not be expected that the spherical harmonic analysis would provide a simple description of the secular change or be helpful in its interpretation.

However, it is a consequence of Gauss's theorem that a field known over a spherical surface and whose origin is internal to the sphere can be produced by an infinite number of different distributions of internal current flow. This can be conveniently demonstrated by supposing that a field, of which the scalar potential is known over a spherical surface of

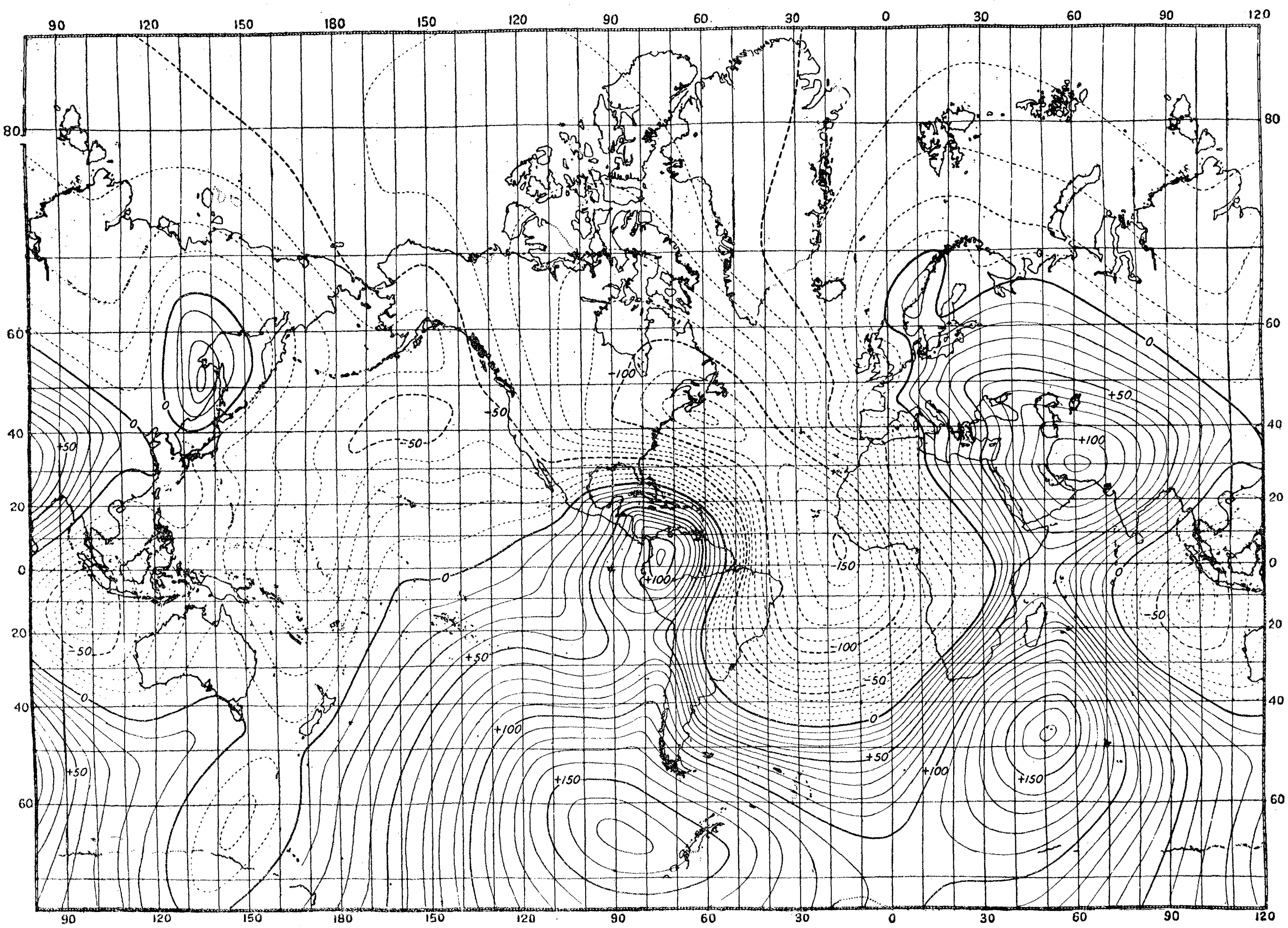

FIGURE 2. Geomagnetic secular change in gammas per year, vertical intensity, epoch 1922.5.

radius r, is to be represented by dipole distributions over an inner spherical surface of radius $a\,(r>a)$. The dipole distribution will be denoted by a polarization vector $\mathbf{M}$, the dipole moment per unit area. There are two cases to consider.

(i) *M radial*. This case is the familiar one of a magnetic shell whose strength is a function of the colatitude θ and the longitude λ. The scalar potential ϕ_1 is given by (1) (Chapman & Bartels 1940, §17·17):

$$\phi_1 = 4\pi \sum_{n=0}^{\infty} \frac{n}{2n+1} M_n(\theta, \lambda) \left(\frac{a}{r}\right)^{n+1}, \tag{1}$$

where M is expressed as the sum of a series of surface harmonics $\Sigma M_n(\theta, \lambda)$.

(ii) *M tangential to the spherical surface.* This distribution is equivalent to a distribution of magnetic poles of density $m = \nabla . \mathbf{M}$. In spherical co-ordinates this gives

$$m = \frac{1}{a \sin\theta} \frac{\partial}{\partial\theta} (\sin\theta \, M_\theta) + \frac{1}{a \sin\theta} \frac{\partial M_\lambda}{\partial\lambda}. \tag{2}$$

Now if m is expressed as a series of surface harmonics $\Sigma m_n(\theta, \lambda)$, the potential ϕ_2 external to the shell is given by

$$\phi_2 = 4\pi a \sum_{n=0}^{\infty} \frac{1}{2n+1} m_n(\theta, \lambda) \left(\frac{a}{r}\right)^{n+1} \quad \text{(Chapman \& Bartels 1940, §17·16).}$$

Thus if the total potential external to the shell is

$$\phi = \phi_1 + \phi_2 = \Sigma\phi_n(\theta, \lambda),$$

then

$$a m_n + n M_n = \frac{2n+1}{4\pi} \left(\frac{r}{a}\right)^{n+1} \phi_n. \tag{3}$$

Thus any magnetic field may be represented either by a suitable distribution over a spherical surface of given radius of radial or vertical dipoles *or* of poles, the latter, in general, being equivalent to a distribution of tangential or horizontal dipoles *either* in the θ *or* in the λ directions.

There are, however, certain exceptions; for instance, a field whose potential is a series of zonal harmonics, functions of θ and r only, cannot be obtained from a distribution of dipoles (M_λ) parallel to lines of latitude, for in this case

$$M_\lambda = \frac{2n+1}{4\pi} \left(\frac{r}{a}\right)^{n+1} \int_0^\lambda \phi \sin\theta \, d\lambda,$$

which is usually not single-valued.

It is therefore necessary to examine whether any restrictions can be attached from the physical point of view which will make the analysis of the secular variation into dipoles and multipoles at points throughout the core unique. It seems reasonable to restrict attention to electric currents as a source of the field; for there is no reason to believe that ferromagnetism can be a property of the liquid core. The current systems, the changes of which give rise to the secular variation, can be thought of as consisting of small closed circuits of current flow. If each of these circuits can be approximated by a current distribution throughout a sphere of radius b, the mathematically simplest distribution of current density (I) which will give rise to a dipole (the source most likely to have an appreciable field at the surface of the earth) is given by

$$I = \frac{3fr^3}{2\pi b^4} \sin\theta, \tag{4}$$

where r is the distance from the centre of the spherical current distribution to the surface of the earth and is taken to be 3500 km. f is the field at the surface of the earth and is likely to be of the order of 0·1 gauss, for $\dot{\mathbf{F}}$ is in many places over 100 γ/year. Now it seems unlikely that the current density required to produce the secular variation is much larger than that required to produce the main field. This can be calculated from equation (4) as 5×10^{-9} e.m.u./cm.^{2}, giving the following inequality for the size of each spherical distribution of current:

$$b > 1400 \text{ km}.$$

Should the current flow in a single loop of this dimension rather than the exact distribution required to produce a dipole field, it can be seen from figures 5*e* and *f* that the field at the surface would be indistinguishable from that of a dipole placed on the axis of the loop and slightly deeper than its centre.

As the dipoles will be separated from one another by 2800 km., or an angular distance of 50°, it can be seen from figures 5*a*, *c* and *e* that at the surface of the earth the field over an appreciable region above each dipole will not be distorted by the other dipoles to a degree which would be noticeable in the data. It seems therefore physically reasonable to search for a representation of the secular variation field in terms of dipoles near the surface of the core sufficiently separated for their fields to be independent, in which case the uniqueness of the identification of the single sources is limited only by the accuracy of the data.

The regional character of $\dot{\mathbf{F}}$, which results from these assumptions and which contrasts strongly with the planetary nature of **F**, is consistent, as was recognized by Bartels (1925), with the smallness of the first harmonic term of the secular variation field.

The problem thus reduces to devising a convenient method of fitting to the secular variation field a set of dipoles placed at points within the core. To compute the x, y, z, components of a field on the surface of a sphere arising from an eccentric dipole with different inclinations is a complicated calculation, and some simple means is required to detect the position of the dipole from an examination of the surface field.

Consider a region of active secular change, and suppose that the field arises from a dipole with the restrictions referred to above. The field vectors at points in any plane passing through the axis of the dipole will lie wholly in that plane. Of such planes only that passing through the geocentre will cut the surface of the earth in a great circle; thus at points on this great circle, and on no other, the horizontal field vectors of the secular change will lie along it. This great circle is most conveniently found by plotting the $\dot{\mathbf{H}}$ vectors on a gnomonic projection of the earth, which has the property that all great circles are represented by straight lines. The part of the great circle in the region where interference from other dipoles is negligible, and which is therefore recognizable on the gnomonic chart, is termed an *S* line. Such projections and *S* lines are shown in figures 3*a* to 3*f*. Having thus found the plane of the dipole, a search may be made for the depth and inclination of the source which best fit the observed magnitude and direction of $\dot{\mathbf{F}}$ for points along this great circle. This is most easily done by trial.

2. The method of plotting $\dot{\mathbf{H}}$ on a gnomonic projection

The gnomonic projection is a geometrical projection from the centre of the sphere on to a tangential plane. The surface of the earth is most conveniently covered by six such charts, with considerable overlap, which are the projections on to mutually perpendicular planes tangent at the poles and equator. These particular projections are not available, but a grid of lines of longitude and latitude at 10° intervals is easily computed and plotted.

The projection is not orthomorphic, i.e. the scales along the lines of latitude and longitude are different and vary over the map. Therefore in order to draw the $\dot{\mathbf{H}}$ vector at a point on these charts, the $\dot{X}$ and $\dot{Y}$ components (tabulated by Vestine *et al.* 1947*a*) are drawn along lines of longitude and latitude to scales proportional to the map scales at that point. The

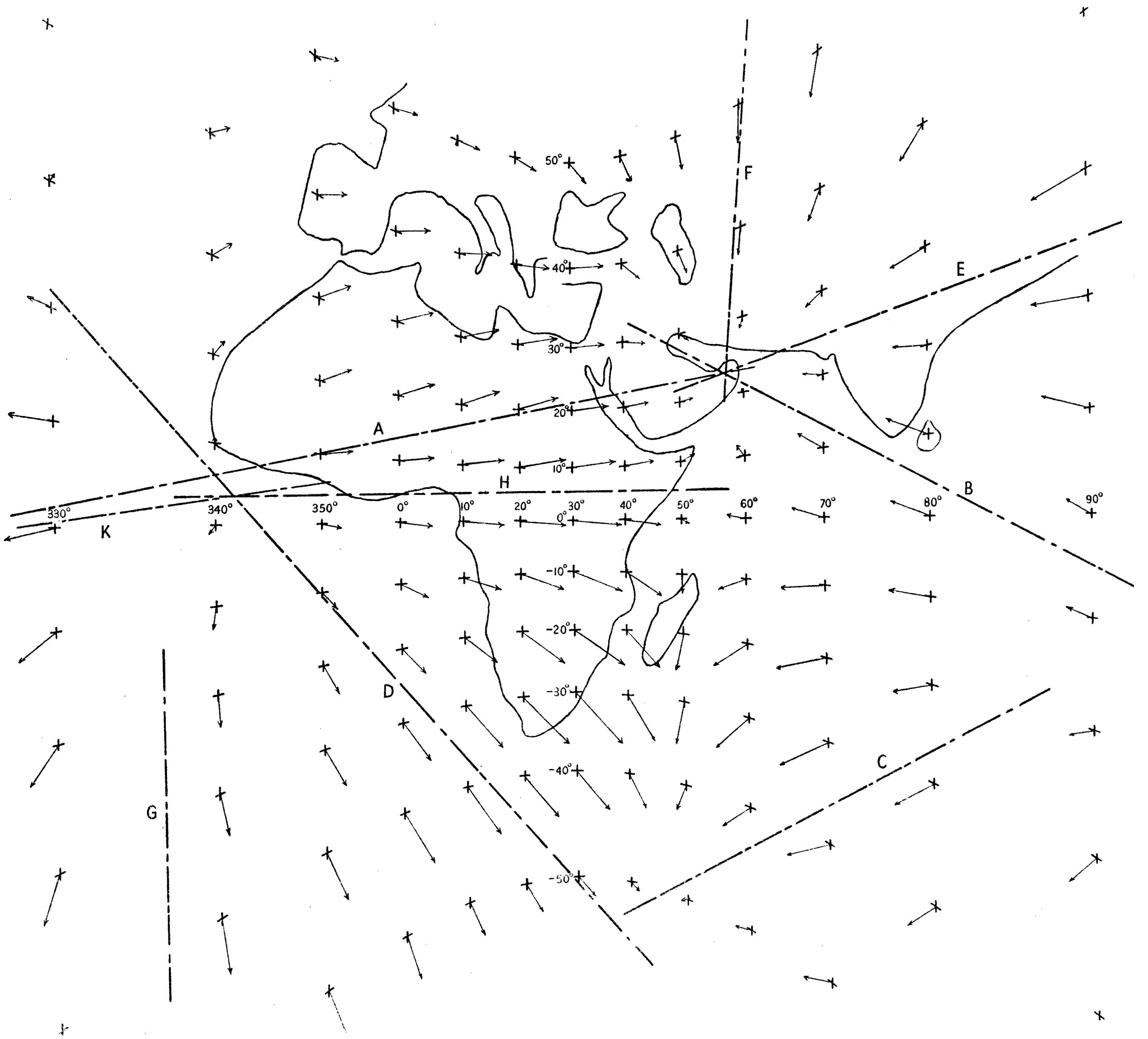

FIGURE 3a

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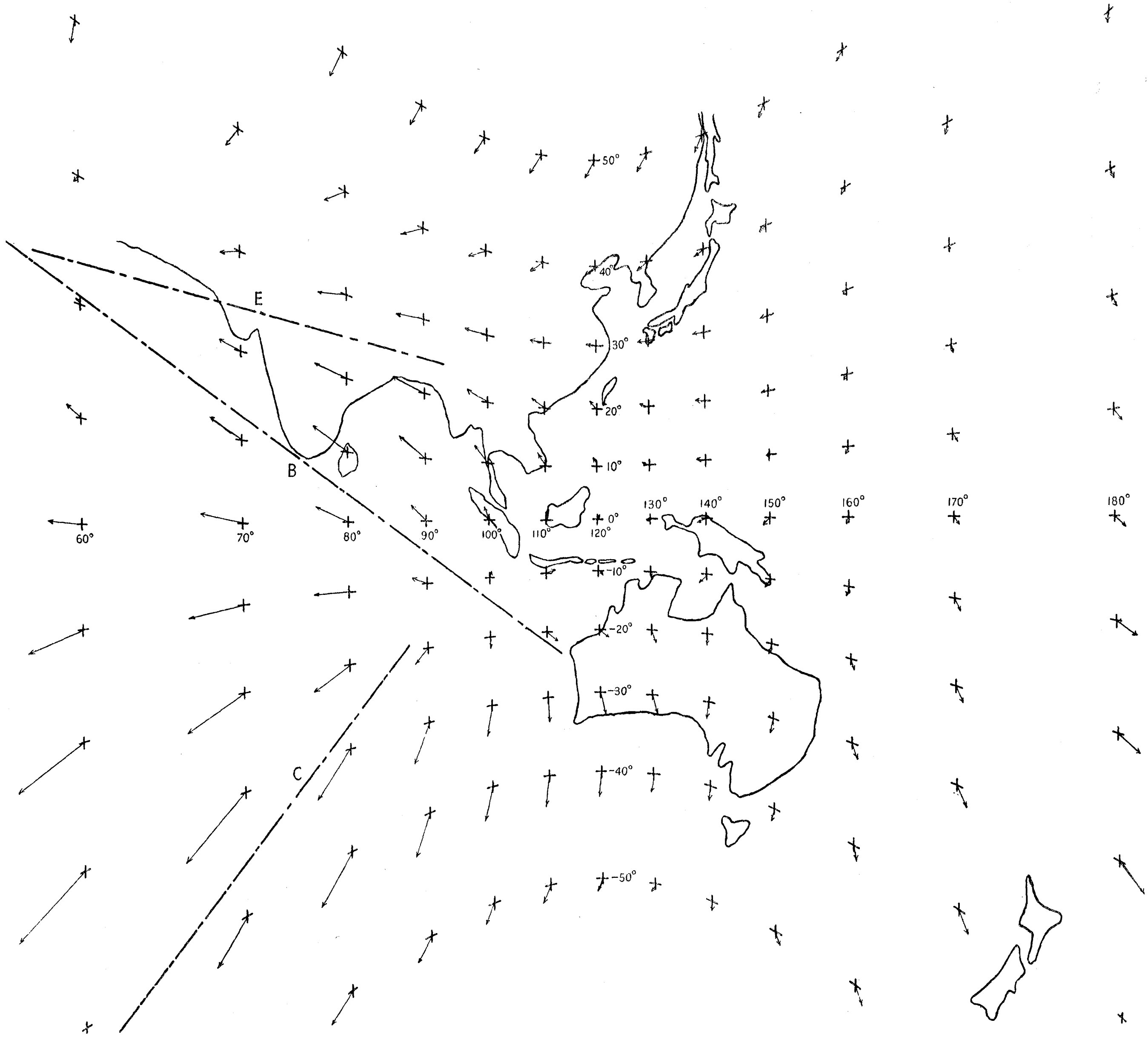
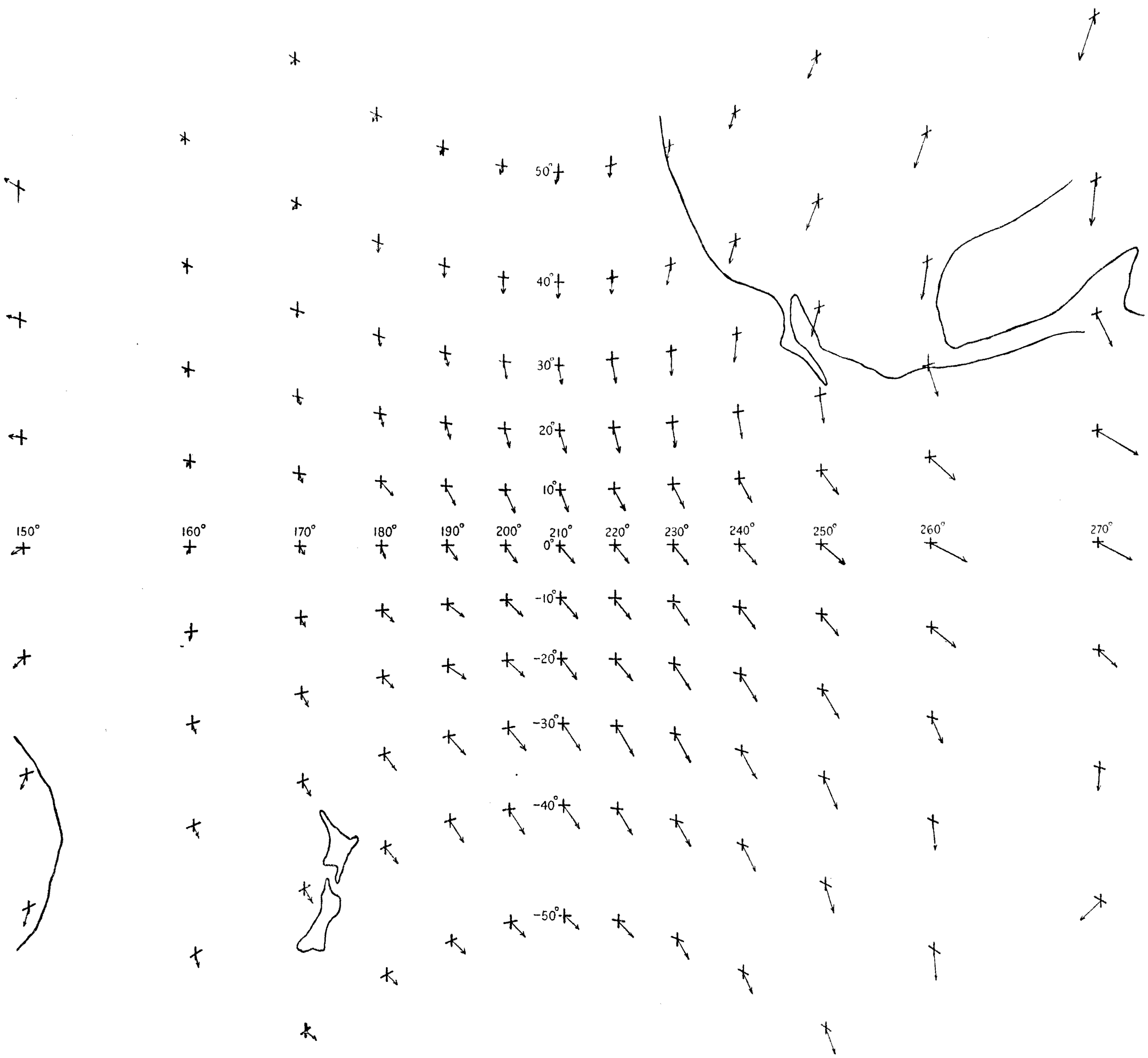

50°
40°
30°
20°
10°
0°
−10°
−20°
−30°
−40°
−50°
60°
70°
80°
90°
100°
110°
120°
130°
140°
150°
160°
170°
180°
E
B
C

FIGURE 3b.



150°
160°
170°
180°
190°
200°
210°
220°
230°
240°
250°
260°
270°
50°
40°
30°
20°
10°
0°
-10°
-20°
-30°
-40°
-50°

FIGURE 3*c*.


50°
40°
30°
20°
10°
0°
-10°
-20°
-30°
-40°
-50°
240°
250°
260°
270°
280°
290°
300°
310°
320°
330°
340°
350°
360°
A
D
G
H
J
K
L
M
N
Q

FIGURE 3*d*.

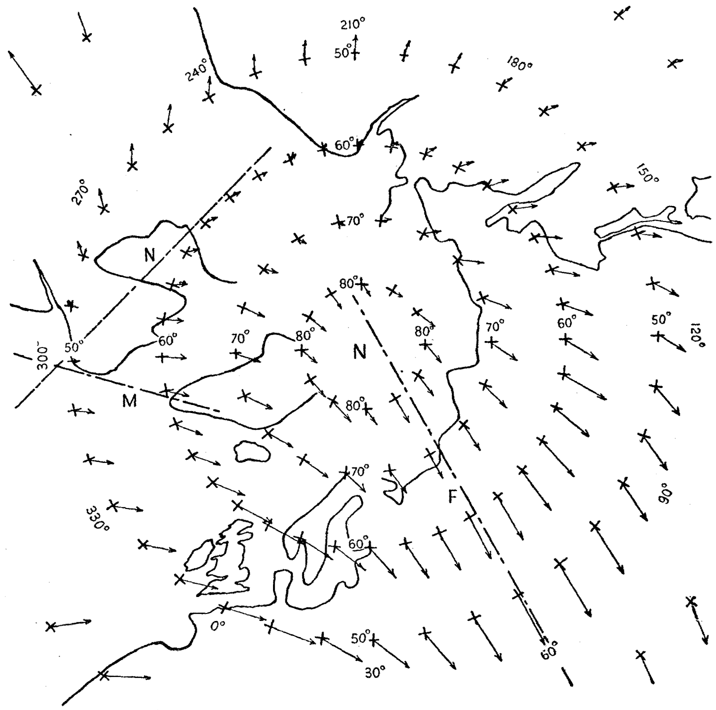


Figure 3*e*.

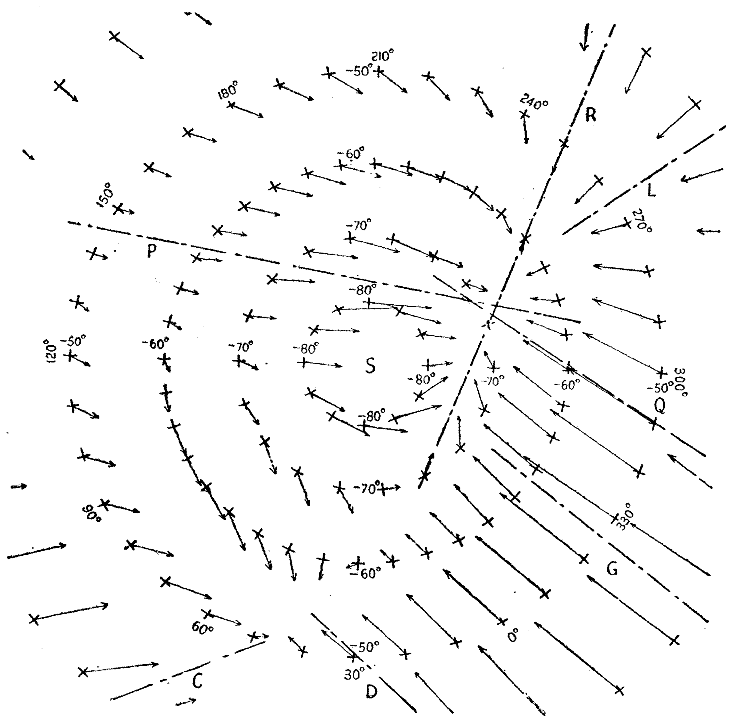


Figure 3*f*.

Figure 3. Horizontal component of secular variation field, epoch 1922.5, on the gnomonic projection, showing the *S* lines found.

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necessary scaling factors can easily be computed for the polar projections but are most conveniently obtained by measurement for the equatorial projections. The scaled $\dot{X}$ and $\dot{Y}$ components are added vectorially to give the $\dot{\mathbf{H}}$ vector. The scale factors are not normalized in the diagrams in this paper, so that the magnitude scale of the vectors varies over the map.

Figures 3*a* to *f* show the vectors of the $\dot{\mathbf{H}}$ field plotted in this way for the epoch 1922.5. It will be seen that there are several *S* lines of appreciable length (greater than 30 to 40°), many of which have the property of joining and intersecting at points of zero $\dot{\mathbf{H}}$, which will

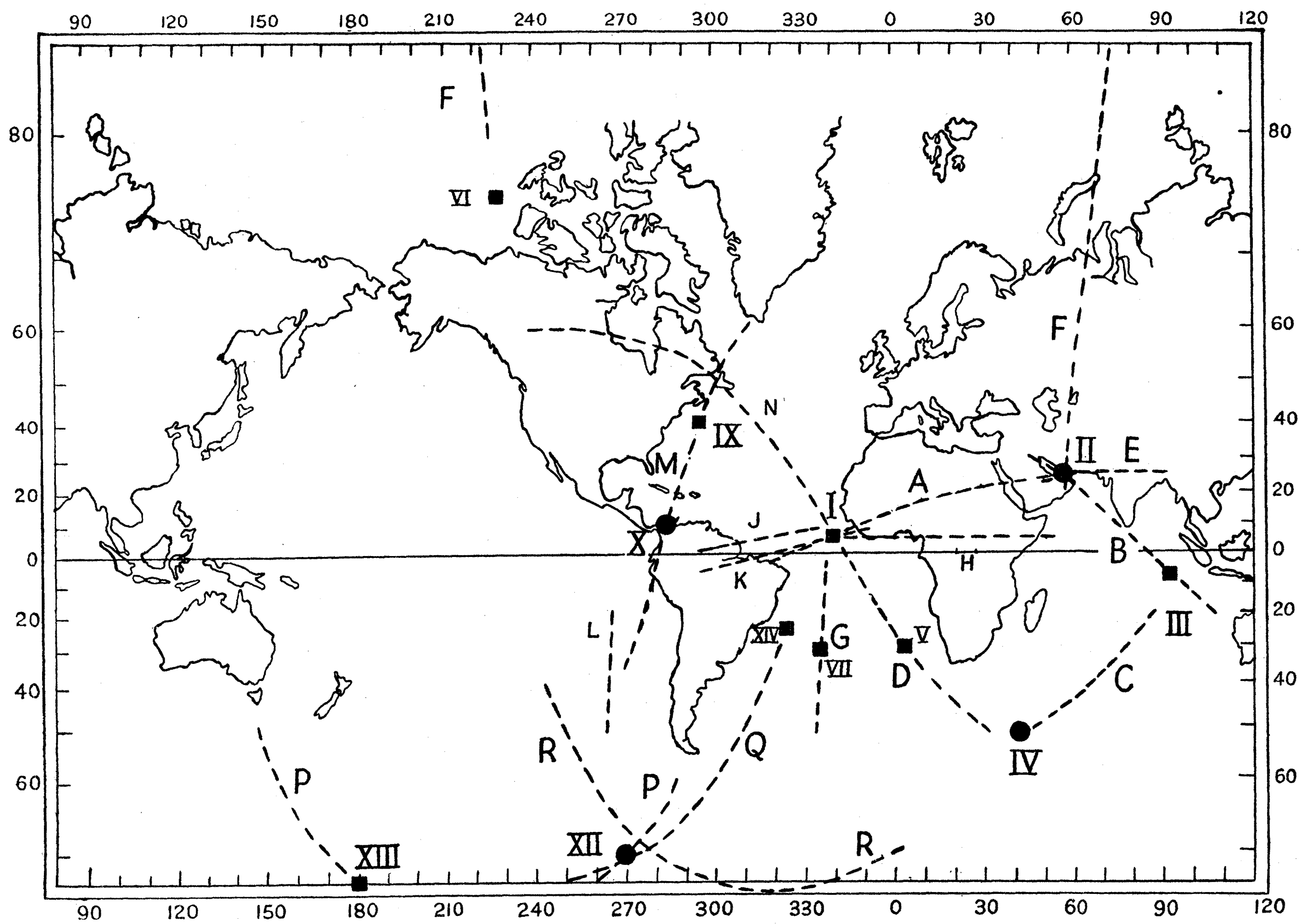

FIGURE 4. Key to sources and *S* sections. ■ denotes a dipole directed radially outwards; ● denotes a dipole directed radially inwards.

be called *S* points. Figure 4 gives an overall picture of the *S* lines and *S* points found. To exhibit the total field $\dot{\mathbf{F}}$ at points along the *S* lines, the $\dot{\mathbf{F}}$ vectors were drawn on diametral sections through each of these lines, the magnitude and inclination (I') of the vectors being computed from the expressions

$$\text{mod}\,\dot{\mathbf{F}} = ((\dot{X})^2+(\dot{Y})^2+(\dot{Z})^2)^{\frac{1}{2}} \quad \text{and} \quad I' = \tan^{-1}\dot{Z}/((\dot{X})^2+(\dot{Y})^2)^{\frac{1}{2}}.$$

These sections, which will be termed *S* sections, are shown in figures 6*a* to *q*. Positions were read to the nearest degree, and true (unscaled) values of $\dot{X}$, $\dot{Y}$, $\dot{Z}$ were obtained by linear interpolation between the 10° grid values.

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3. Interpretation and fitting of S section fields

It was stated above that the sources lying in the S planes are likely to lie beneath the surface of the core and not too deep within it. The object of the analysis was to find the simplest interpretation of the secular variation field in terms of simple sources at a depth below the earth's surface of about $0{\cdot}5R$, where R is the radius of the earth (the core boundary is at depth $0{\cdot}45R$). An extensive series of fields was obtained and the field vectors drawn

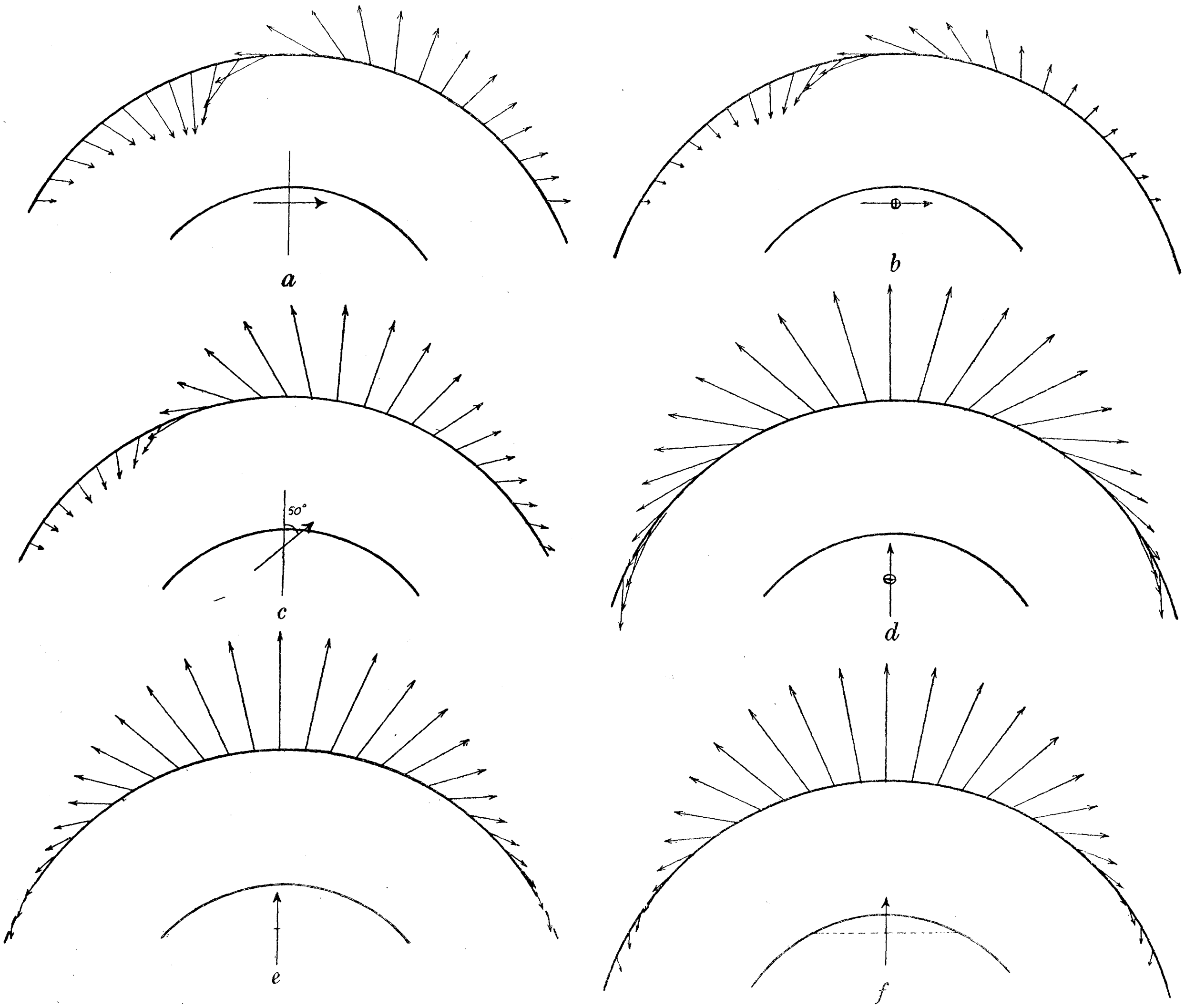


Figure 5. Theoretical field distributions: *a*, horizontal dipole at depth $0{\cdot}5R$; *b*, horizontal line of dipoles at depth $0{\cdot}5R$; *c*, inclined dipole at depth $0{\cdot}5R$; *d*, vertical line of dipoles at depth $0{\cdot}6R$; *e*, vertical dipole at depth $0{\cdot}6R$; *f*, current loop, radius $0{\cdot}25R$, at depth $0{\cdot}5R$.

at 5° intervals over an angular distance of $\pm 60°$ on diametral sections through the source. In particular, the fields were computed for dipoles at depth $0{\cdot}5R$ with inclinations to the vertical of 0°, 10°, 20°, ..., 90° and for vertical dipoles at depths $0{\cdot}4R$, $0{\cdot}6R$ and $0{\cdot}7R$. To give an indication of the effect of finite extension of source, a similar series for an infinite line of dipoles at right angles to the S plane was computed and drawn. The magnetic fields at the surface of the earth due to large current loops of radius $0{\cdot}15R$, $0{\cdot}25R$ and $0{\cdot}40R$ at the surface of the core were also obtained. This was done experimentally rather than by computation, as two-dimensional scale experiments using one apparatus permit the investigation of current loops of different size, while the lengthy process of computation would have to be repeated for each size.

With the imposed geometrical configuration and the required accuracy of 1 % it was realized that greater flexibility and convenience would be obtained by using alternating current and amplification rather than the conventional d.c. ballistic methods, and it was more convenient to use a null method than measure the very small voltages induced in the pick-up coil.

The current loop in the experiment was a single vertical turn of copper strip carrying 100 A 50 c./sec. current. The measuring element was clamped to an arm pivoted at one end, the surface of the earth being represented by the horizontal circle swept out by the measuring element as the arm was rotated, the radius being about a metre in this experiment. Different configurations were obtained by changing the radial scale or the size of the current loop.

The measuring element consisted of two vertical coaxial coils (the pick-up coil of 500 turns and the nulling coil of 100 turns) which could be rotated about a vertical axis passing through the centre of the coil system. At any one position the coil system was rotated until, with no nulling current, the induced e.m.f. in the pick-up coil was zero, or a minimum, as indicated on an oscilloscope, using a low-frequency amplifier with a gain of about 10^7. The plane of the coils then gave the field direction, and the angle relative to the radial direction was read on a circular scale. After rotating the coils through 90°, the current in the nulling coil, which was fed from a resistive tap in the main current circuit, was adjusted to give again a minimum signal on the oscilloscope. This current, as determined by the total resistance in the nulling circuit, gave a measure of the relative field intensity.

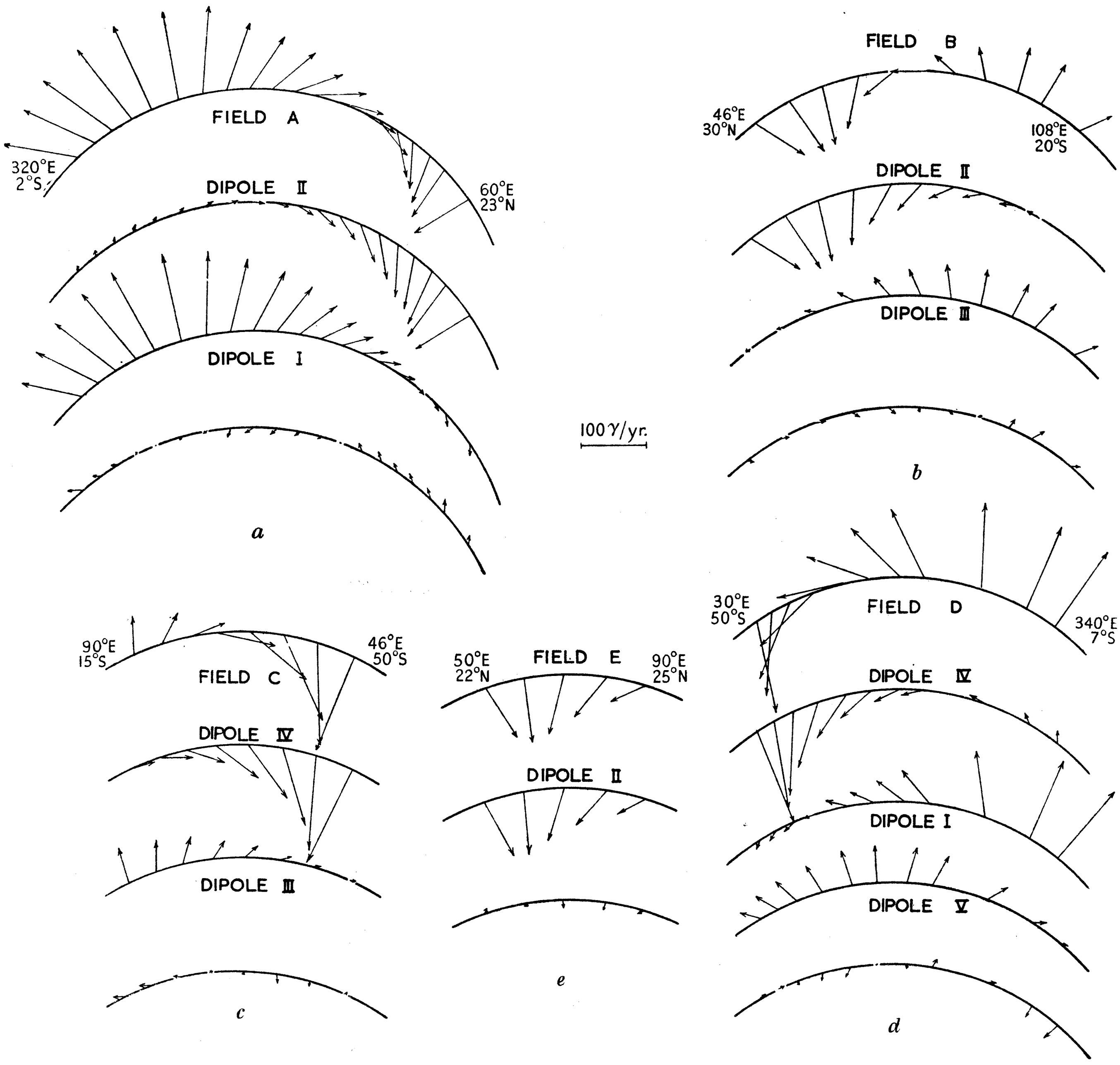
By careful siting of the apparatus in an isolated hut, stray 50 c./sec. field was avoided, and the use of this frequency entailed no great difficulties, while very much simplifying the apparatus. The accuracy was mainly determined by the precision of the positioning and shape of the current loop, and an error of less than 1° in direction and 1 % in intensity was easily obtained in most measurements.

Examples of the various fields obtained by computation and by experiment are given in figures 5 *a* to *f*.

It is found by trial that no *S* section can be fitted by any one source, a general feature being the approximate constancy of the magnitude of the $\dot{\mathbf{F}}$ vector along a large part of most of the sections, a property which no known point source possesses. Nor would the *S* lines in practice be expected to intersect if each were due to a source in the middle of the line, for they would in such circumstances terminate before the region in which the fields of the two sources became of comparable magnitude.

It is conceivable that two more or less parallel inclined sources could give a parallel *S* line passing between them, but again it is not possible to see how two or more lines could intersect, or how any fairly simple distribution of such sources could produce the observed field. No doubt individual sections could be explained by the addition of two or more inclined sources lying in them, but this would increase the difficulty of explaining the intersections, as it would bring the sources closer together.

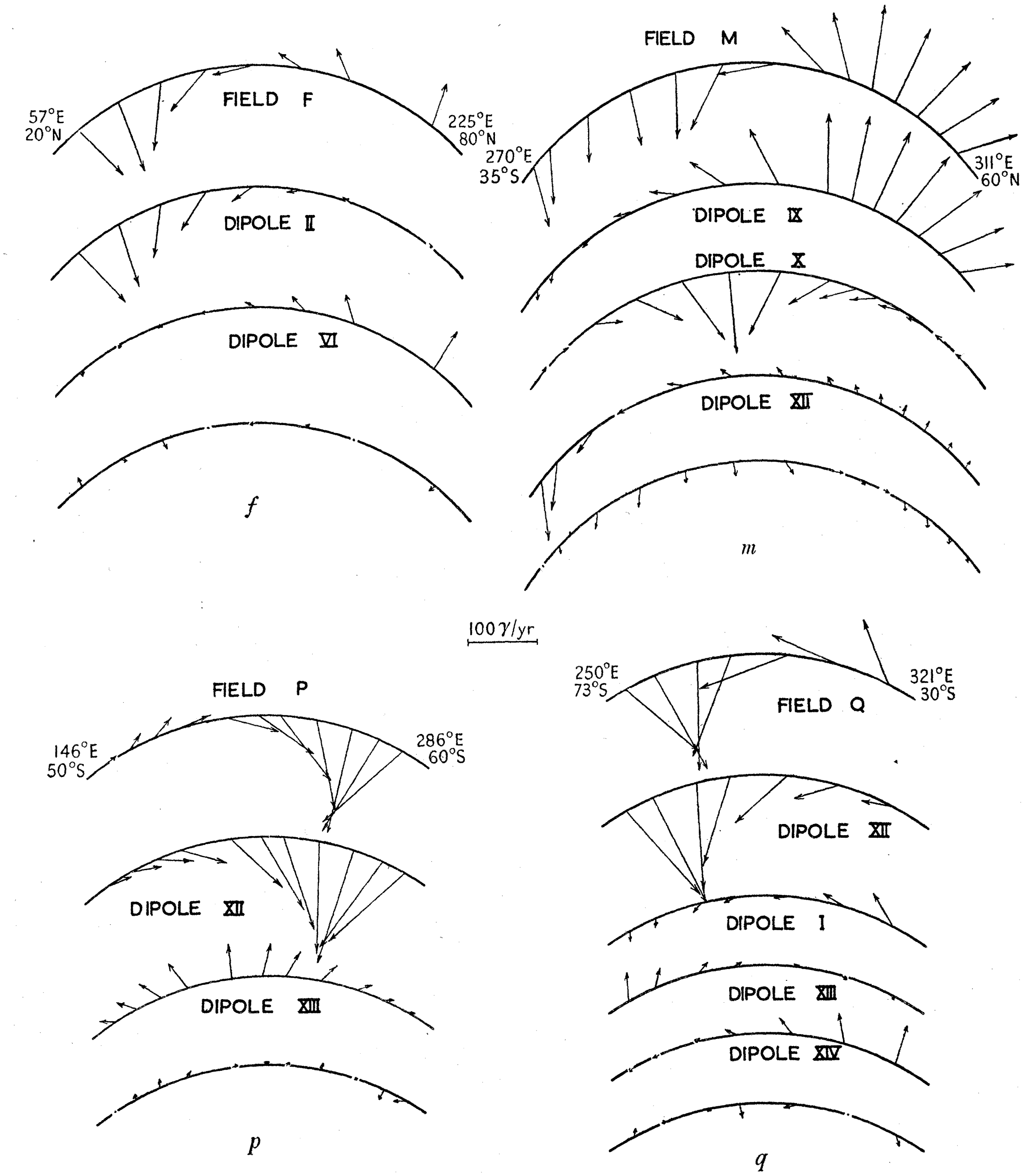
It is therefore concluded that the only possible solution is that of vertical dipoles under the various S points. Two vertical dipoles would certainly produce an S line joining them, and the addition of further sources would not noticeably distort this field as the sources are



FIGURES 6*a* to *e*.

on the whole widely separated (50 to 60°). At this distance, the surface horizontal field of a vertical dipole at depth $0{\cdot}5R$ is only 10 % of the maximum field, and in the placing of the S lines horizontal fields of less than 10γ/year are not really significant.

The problem is then greatly simplified, as the position, inclination and approximate magnitude of several of the sources are known, and the analysis is reduced to determining the set which best fits all the S sections. It is found that dipoles give the best fit, and that this fit is improved if they are placed at a depth $0 \cdot 6R$ below the earth's surface rather than at the depth $0 \cdot 5R$ which was initially tried. The identification, particularly as regards depth,



FIGURES 6*f*, 6*m*, 6*p*, 6*q*.

FIGURE 6. The fitting of *S* sections. The top section gives the observed field, then the approximating dipole fields are given, the bottom section giving the residual vectors (observed minus all approximating fields).

is more certain for the larger sources and for those cases where the *S* section extends up to and through the source and is easier if the dipoles at the ends of each of the *S* sections have opposite polarity.

The actual method of fitting is as follows. As described in §2 the $\dot{\mathbf{F}}$ vectors are drawn at varying distances (5 to 10°) along the *S* sections as shown in the top sections of figure 6. From these observed field vectors are subtracted the field vectors of that radial dipole under an *S* point which most closely fits the field near that point. This leaves a set of vector field differences along the *S* line which is the starting point of the next similar process until, after the subtraction of two (in some cases three) dipoles, whose fields are shown in the middle sections of figure 6, a set of residual vectors is left which are shown in the bottom sections of figure 6. By trial and error the best magnitudes and depths of the dipoles can be obtained and systematic residuals eliminated, the final choice of magnitude being made by comparing the mean value of the residuals obtained when the dipole strengths are varied by $\pm 5\gamma$/year.

The residuals finally left are random, but of appreciable magnitude 10 to 16γ/year, and it is interesting to estimate the overall random deviation introduced by the method of fitting. Independent random errors are introduced in each of the stages of computation, drawing and subtraction of the various fields. The largest are those involved in the interpolation between the 10° grid values, the drawing of the approximating fields from the comparison dipole fields, and the vector subtraction. Some can be estimated from repeated drawings, and by assigning reasonable values to the others a final r.m.s. deviation of 8γ/year is obtained; i.e. if all the drawings were repeated the average magnitude of the difference vectors between the two sets of residuals would be expected to be 5 to 10γ/year.

Vestine, Laporte & Cooper (1946) estimate that the accuracy of their charts is 10γ over many land areas and 15 to 20γ over most oceanic and polar areas. The average residuals obtained from all the *S* sections is 14γ/year, and this is taken to indicate as good a measure of fit as can be expected from the data available. The average magnitude of the secular change field over the world is about 70γ/year, and over the area covered about 80γ/year.

4. The sources found

In figures 3, 4 and 6 and this discussion, the *S* lines and sections are lettered A to Q and the sources are numbered I to XIV. The secular variation fields, the approximating dipole fields and the residual fields for the main sections are all given in figures 6*a* to *q*.

The closed circuit of *S* lines A, B, C, D is particularly susceptible to fitting, giving vertical dipoles I, II, III, IV at the *S* points, with lines H, J, K, N and E, F supporting the identification of I and II respectively. The only complication is the necessity to add the vertical dipole V in D. The field of this is definitely present, but as it is rather small compared with that of the two dipoles at the ends of the *S* line it is not possible to say that the source must be exactly under it. It is in this area round Africa that the method proves most valuable, and there is a very high degree of identification of the four main dipoles. Having found that vertical dipole sources give a close fit to the observed field, it is natural to look first for vertical dipoles in other sections, where they are not demanded by intersections, but trial shows that the use of inclined dipoles does not give a better fit.

Section F indicates a vertical dipole VI in the north polar region, but, as there is no confirmation by another section, no reliance can be placed on its identification. H, J, K and L are examples of sections which do not pass through or very near a second source, but are due to superposition of two or more fields. They cannot be relied on to give more than an indication of the nature of a second source, and it is significant that they are all comparatively short. G needs two other vertical dipoles VII and VIII to fit it, but as with F no real identification can be claimed. It is very likely that G belongs to the same class as H, J, K and L, in which case VII and VIII are meaningless.

The American region is much more difficult to explain—the solution eventually arrived at is that of two vertical dipoles IX and X in M, though their identification is not very good. These are so close together that the sections N and J, K which would be expected to pass through the sources have been pushed apart. Also as N is not completely fitted, there must be some unidentified source XI further west. There is definitely a vertical source XII at the intersection of P, Q and R, but while in P and Q it appears to be a dipole at depth 0·6*R*, no fit can be obtained in R. However, together with the small vertical dipole XIII needed to fit P and Q, XII is in the south polar region, where observational data is very scarce and Vestine's figures are less accurate. To complete the picture a small vertical dipole XIV is needed to fit Q.

Tables 1 and 2 summarize the fit obtained in the main sections and the identification of the sources.

TABLE 1. *S* SECTION DATA

S section	dipoles in section	length of *S* section (degrees)	number of $\dot{F}$ vectors drawn	r.m.s. magnitude of residual vectors (γ/year)
A	I, II	100	20	15
B	II, III	80	11	16
C	III, IV	45	8	15
D	IV, V, I	70	11	16
F	II, VI	80	8	11
M	IX, X, XII	100	14	17
P	XII, XIII	65	11	11
Q	XIII, XII, XIV, I	55	7	13

TABLE 2. IDENTIFICATION OF SOURCES

source (2)		depth as a fraction of *R*	position (3)	maximum surface field (γ/year) (1)	
I	vertical dipole	0·6 ± 0·05	341° E, 5° N ± 3°	135 ± 5	definitely identified
II	vertical dipole	0·6 ± 0·05	57° E, 23° N ± 3°	100 ± 5	definitely identified
III	vertical dipole	0·6 ± 0·05	92° E, 7° S ± 3°	60 ± 5	definitely identified
IV	vertical dipole	0·6 ± 0·05	46° E, 50° S ± 3°	160 ± 10	definitely identified
VI	vertical dipole	0·6 ± 0·1	210° E, 75° N ± 5°	65 ± 10	reasonable identification
IX	vertical dipole	0·6 ± 0·1	295° E, 40° N ± 5°	125 ± 10	reasonable identification
X	vertical dipole	0·5 ± 0·1	284° E, 10° N ± 5°	120 ± 10	reasonable identification
V	vertical dipole	0·6 ± 0·1	4° E, 30° S ± 3°*	60 ± 5	existence as separate sources doubtful; see note (4)
VII	vertical dipole	0·6 ± 0·1	335° E, 30° S ± 5°*	60 ± 10	existence as separate sources doubtful; see note (4)
VIII	vertical dipole	0·6 ± 0·1	327° E, 75° S ± 5°*	140 ± 10	existence as separate sources doubtful; see note (4)
XIV	vertical dipole	0·55 ± 0·1	323° E, 25° S ± 5°*	60 ± 5	existence as separate sources doubtful; see note (4)
XI	unidentified	—			
XII	unidentified		270° E, 70° S ± 3°	170 ± 10	in south polar region
XIII	vertical dipole	0·55 ± 0·1	180° E, 73° S ± 5°	50 ± 5	in south polar region

Notes

(1) A surface field of 100γ/year implies a dipole at depth 0·6*R* of strength $2{\cdot}78 \times 10^{22}$ gauss cm.^{3}/year.

(2) Possible deviations from the vertical are about ±2° in I, II, III, IV and XII, about ±5° in IX and X, and about ±10° in the others.

(3) Errors in position are only rough estimates. When starred (*) they refer only to the plane of the section.

(4) It is likely that dipoles V, VII and XIV in D, G and Q are really an indication that on its southern side I is not a simple dipole. All that can be said here is that in terms of simple sources the solution obtained is valid.

5. Comparison with other analyses

Other analyses of the secular variation field are the world-wide one by McNish (1940) and that of the South African region by Bullard (1948).

Bullard, using the data for epoch 1922.5 given by Vestine *et al.* (1947*a*), attempted to explain the secular variation field along a cross-section in the South African region by postulating an approximately horizontal dipole at 20° E, 25° S at a depth about $0{\cdot}5R$. This result has not been confirmed in the present work and its validity cannot be accepted for

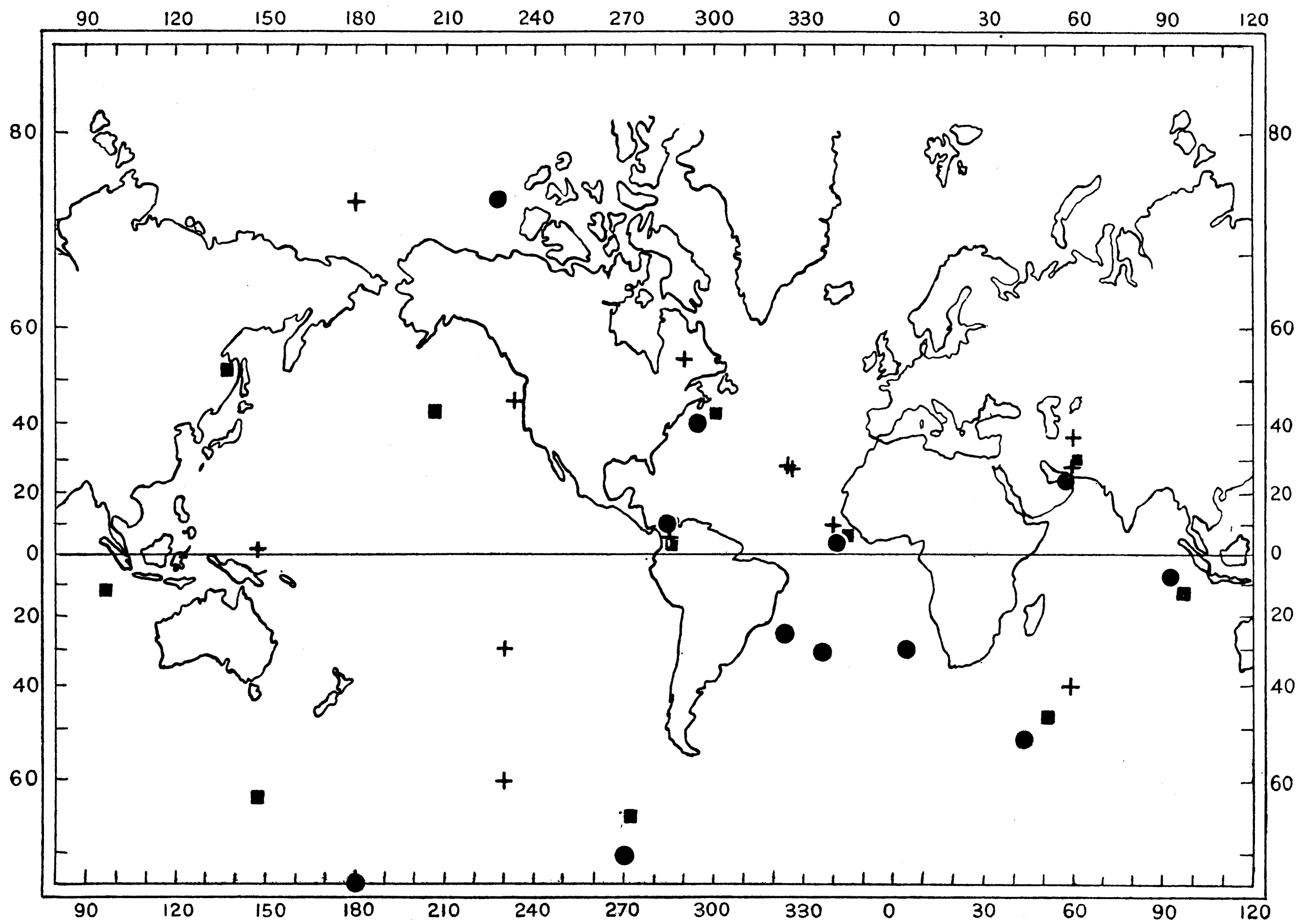


Figure 7. Positions of dipoles postulated by McNish (1940) and the authors, and the isoporic foci in vertical intensity obtained from Vestine (1947*a*), all for epoch 1922.5. +, McNish; ●, authors; ■, Vestine.

several reasons. Though the section drawn was a straight line on the map projection used, it was not even approximately a great circle, as the method of fitting assumed it to be. In the section chosen the fit obtained with a horizontal dipole is not a good one, the average residual being about 30γ/year and the residual vectors are decidedly non-random, for the observed field is greater and less steeply inclined than the dipole field at the ends of the section. Bullard states that this discrepancy can be removed by substituting a finite line of dipoles for the point dipole, but though the extension of the source into a line would give a better fit in direction, the magnitude of the field would fall off even more rapidly towards the ends of the section, as can be seen by comparing figures 5*a* and *b*. When a much longer great circle section of the field in this vicinity is obtained (figure 6*d*) it is easy to see that no horizontal point source could account for any major part of it.

McNish, using an isoporic chart in vertical intensity stated to be of approximate epoch 1920–25, attempted to explain the vertical field (excluding the polar regions) by means of vertical dipoles placed at depth $0{\cdot}5R$ vertically below the centres of maximum variation. He was able to make all the thirteen dipoles he used the same strength ($1{\cdot}37\times 10^{22}$ gauss cm.^{3}/year, equivalent to a maximum surface field of 85γ/year), placing some close together to explain the larger and more extensive centres. He does not give a numerical estimate of the fit obtained, but describes it as 'thoroughly satisfactory'. His conclusion that the secular variation field can be represented by vertical dipoles is the same as that of the present paper. However, since McNish started with the assumption that the dipoles were vertical and made no attempt to see if inclined dipoles would fit the field his argument is not entirely conclusive. In the present work the authors were forced to the conclusion that many of the sources *must* be vertical dipoles.

There are marked discrepancies between the dipoles found in the two analyses (see figure 7), and these must be attributed to the field data used; the isoporic chart is noticeably different from the $\dot{Z}$ chart for epoch 1922.5 given by Vestine. McNish gives no information as to the source of his data, but he presumably used the isoporic charts of Fisk (1931), and Vestine *et al.* (1947*a*) has pointed out several deficiencies in the method of preparation of these charts which led to reduced accuracy. Vestine had available much more observational data, and systematically eliminated extraneous variations, so it is certain that the data used in this analysis are the more accurate.

The present work has definitely identified four of the active areas in vertical intensity as due to vertical dipoles, with reasonable identification as vertical dipoles of two others, leaving unidentified only two small foci and one in the south polar region. The positions of the dipoles are close to, but do not coincide with, the isoporic foci of Z (figure 7), as the points of zero $\dot{\mathbf{H}}$ do not exactly coincide with positions of maximum $\dot{Z}$. The average angular distance between dipole and corresponding focus is about 5°, giving a measure of the possible positional error of the dipole which agrees well with those (3 to 5°) estimated from the fitting of the sections.

6. Physical interpretation of the results

The vertical magnetic dipoles found by this analysis will in fact be horizontal circuits of current flow, the current distribution depending on the depth at which the circuit is placed. It is found that finite current loops can be approximated by dipoles placed deeper in the earth, e.g. compare figures 5*e* and *f*, and table 3 gives the depth of a vertical dipole whose field distribution at the surface of the earth approximates to that of a horizontal circular current loop at the surface of the core.

Elsasser (1950) considers that the currents which produce the secular variation must originate in a very thin shell at the surface of the core. Adopting this view, table 3 shows that the actual current systems corresponding to each dipole of the analysis are likely to be about 3000 km. in diameter or 52° in angular distance, which is quite consistent with the observed spacings between the sources. The actual current distribution of the system can be computed by spherical harmonic analysis; the convergence is slow, but the first seven terms indicate an effective diameter considerably less than this. Elsasser's view is based on estimates of the conductivity of the material of the core which is assumed to be molten iron. He takes its conductivity (σ) as 3000 ohm^{-1} cm.$^{-1}$, and calculates the thickness of that outer shell of the core which will attenuate magnetic field fluctuations of a hundred years' period in the ratio 1 : e. He concludes that the magnetic effect of such changes at greater depth would be entirely negligible. Bullard (1950), quoting unpublished work of Fullerton, gives a value of 5000 ohm^{-1} cm.$^{-1}$ which we adopt, giving the skin depth as 40 km.

TABLE 3. DEPTH OF DIPOLE WHOSE FIELD APPROXIMATES TO THAT OF CURRENT LOOP AT SURFACE OF CORE

best depth of dipole (R)	diameter of current loop (km.)
0·5	1,800
0·6	3,000
0·8	5,000

The result that vertical dipoles or horizontal current loops have been found in the above analysis can be understood on this hypothesis that the currents which produce the secular variation are restricted to a thin skin of the core. The horizontal dipole components will be minute compared to the vertical ones, for the dipole strength is the product of the current flow and the area of the current circuit, and the horizontal areas of the skin are very great compared with the cross-sectional areas in the skin. As the areas of the current circuits of vertical and horizontal dipoles are in the ratio of about $(3000)^2$ to $(40)^2$ it is easily seen that only vertical dipoles are likely to have appreciable magnitude.

7. RELATION OF THE ANALYSIS TO THE THEORIES OF THE SECULAR VARIATION

Elsasser (1946) and Bullard (1948) have developed the theory that the secular variation is due to changes in the currents induced in moving matter in the earth's core by the earth's main magnetic field. Elsasser estimates the velocities of the motion in two ways:

(i) In his 1939 paper he assumed that a part of the fluid must move a distance comparable to the linear dimensions (1000 km.) of an active area in the lifetime of the area (100 years). This velocity is 0·03 cm./sec.

(ii) He pointed out (1949) that the westerly drift of the isoporic foci corresponds to a motion of 0·1 cm./sec. at the surface of the core.

Elsasser then pointed out that a velocity $\mathbf{v}$ of this order generates by electromagnetic induction currents of density $\mathbf{i} = \sigma \mathbf{v} \times \mathbf{B} \times 10^{-8}$ A/cm.^{2}, where $\mathbf{B}$ is the earth's field near the surface of the core. Taking $B = 3$ gauss this current density works out at about 10^{-6} A/cm.^{2}, which seems capable of producing the secular variation, as the main field could be produced by a current density of 5×10^{-8} A/cm.^{2}, assuming the simplest distribution throughout the core. However, Bullard's attempt (1948) to explain the distribution of the secular variation field in South Africa by an induced horizontal dipole just below the surface of the core, which varies in magnitude but not in direction, indicated a quantitative difficulty. He proposed that the fluid motion causing the dipole was a spherical or cylindrical eddy just beneath the core surface, rotating about a horizontal axis directed NW–SE, but showed that this induction process did not give a big enough secular change field for a reasonable size of eddy assuming a main field of 3 gauss at this depth.

In general, the disturbance to a field caused by a rotating conductor cannot exceed that produced when the inducing field inside the material is just cancelled by the field due to the induced currents. These currents then flow in the surface of the rotating material. In such a case the material becomes effectively diamagnetic and the dipole moment of the induced field becomes equal to $-3V\mathbf{F}/8\pi$, where V is the volume of the rotating material and $\mathbf{F}$ the component of the inducing field (supposed uniform) perpendicular to the axis of rotation. If the material is a sphere only the first harmonic of the induced field is present, and if the material is an infinite cylinder the external induced field is such as would be produced by a line of dipoles along the axis of the cylinder. In both cases the dipoles are directed anti-parallel to $\mathbf{F}$. Assuming that a centre of secular change persists for about a century, the analysis implies that there should be about twelve dipoles, each of approximate magnitude 3×10^{24} gauss cm.^{3} formed by induction within the core, their variation in magnitude producing the secular variation field. Assuming that the field at the surface of the core is about 3 gauss, we can calculate the total volume of the eddies required to give all the dipoles; it is about 10^{26} cm.^{3}, over one-half of the volume of the core.

To overcome this difficulty of the inadequacy of the field, Bullard (1949) proposed that relative rotation between outer and inner parts of the core produces a large toroidal field from the dipole field by induction. He showed that the toroidal field lies entirely within the core, vanishes at the boundary and can be quite large, i.e. of the order of 50 gauss, for quite small linear velocities of the order of that required to explain the westerly drift. He proposed that such an amplified field would be sufficient to remove the difficulties mentioned above. The toroidal field will induce in an eddy rotating sufficiently quickly a dipole anti-parallel to this field, and thus the secular variation pattern should be explicable in terms of horizontal rather than vertical dipoles at the surface of the core if the eddies were entirely within the skin. From figure 2 the positions for the epoch 1922.5 of the isoporic foci in Z and the lines of zero change can be seen. Bullard (1949) suggests that the pattern might be explained by a line of horizontal dipoles beneath this line of zero change. The corresponding fluid motion would be some type of long cylindrical eddy following this line. The present analysis does not support this suggestion.

If the explanation of the observed vertical dipoles in terms of the skin effect for a highly conducting core is true, the situation is complicated, and it is possible that changes in large horizontal dipoles might produce vertical secular change dipoles. However, the objections given in the next paragraph still apply.

The toroidal field produced by the relative motions of the core is zero at the equator and varies as the sine of twice the angle of latitude. Thus the toroidal field would not help to produce secular variation dipoles very near the equator. Yet the above analysis shows that these exist. The poloidal fields that Bullard (1949) postulates for a dynamo theory of the main field do not vanish at the equator, but these cannot have a magnitude much in excess of the dipole component of the main field, which as we have seen is not sufficient to explain the vertical dipoles of the secular variation.

8. Conclusions

It cannot be guaranteed that there are no undiscovered sources as the method of analysis can only be used in favourable regions, but there are certainly no other large sources. It would be interesting but laborious to repeat the analysis for other epochs, but there is no reason to believe that essentially different results would be obtained, and the fact that the dipoles are vertical makes it possible to obtain their approximate position for other epochs by inspection of the isoporic charts in Z, which confirms that there are generally sources very near the equator.

The interpretation of the results in terms of a two-dimensional current flow in a thin spherical shell at the surface of the core gives physical significance to the representation of Vestine *et al.* (1947*b*) of the secular variation field by a spherical surface distribution of current.

The presence of several vertical dipoles near the equator does not support the assumption of a large toroidal field within the core (as this is zero at the equator), which is an essential feature of Bullard's (1949) dynamo theory of the main field and which arose out of his work on the secular variation. The results of this analysis show that no existing theory of the secular variation is satisfactory.

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